MATEMATIKA 1 – 2. KOLOKVIJ – GRUPA C 17.12.05.

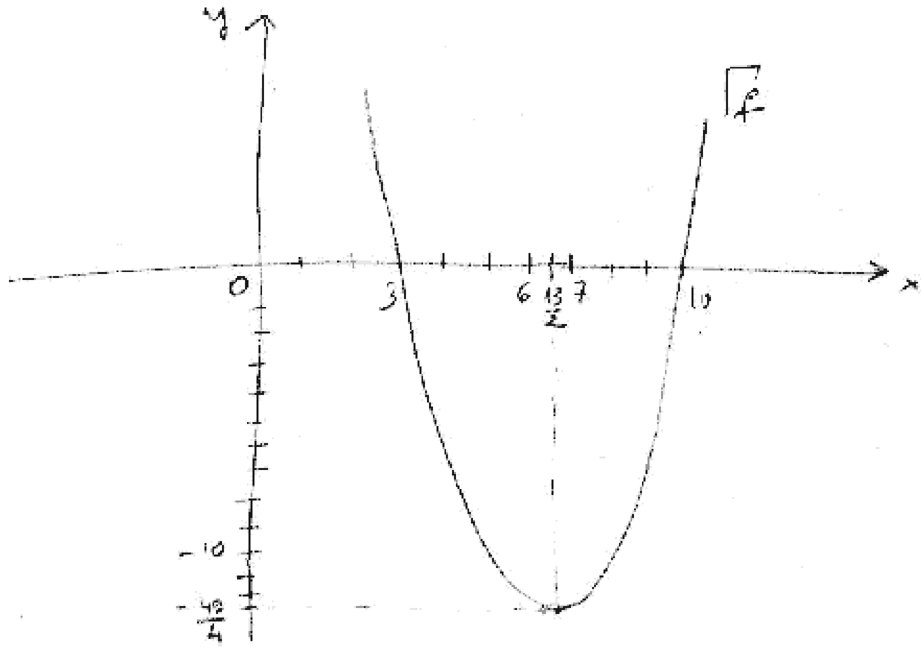
Zadatak 1. (i) $f(x) = x^2 - 13x + 30$

$D = 169 - 120 = 49 \Rightarrow \quad x_{1,2} = \frac{13 \pm 7}{2} \Rightarrow \quad x_1 = 3$

$x_2 = 10$

$T = \left(\frac{-b}{2a}, \frac{-D}{4a}\right)$

$T\left(\frac{13}{2}, -\frac{49}{4}\right)$



(ii) $\mathcal{R}(f) = [-\frac{49}{4}, \infty\rangle$, lok. minimum $\left(\frac{13}{2}, -\frac{49}{4}\right)$

(iii) $4^x - 13\,2^x + 30 = 0$

$2^x = t \Rightarrow \quad t^2 - 13t + 30 = 0$

$t_1 = 3 \Rightarrow \quad 2^x = 3 \,/\log_2 \Rightarrow \quad \boxed{x_1 = \log_2 3}$

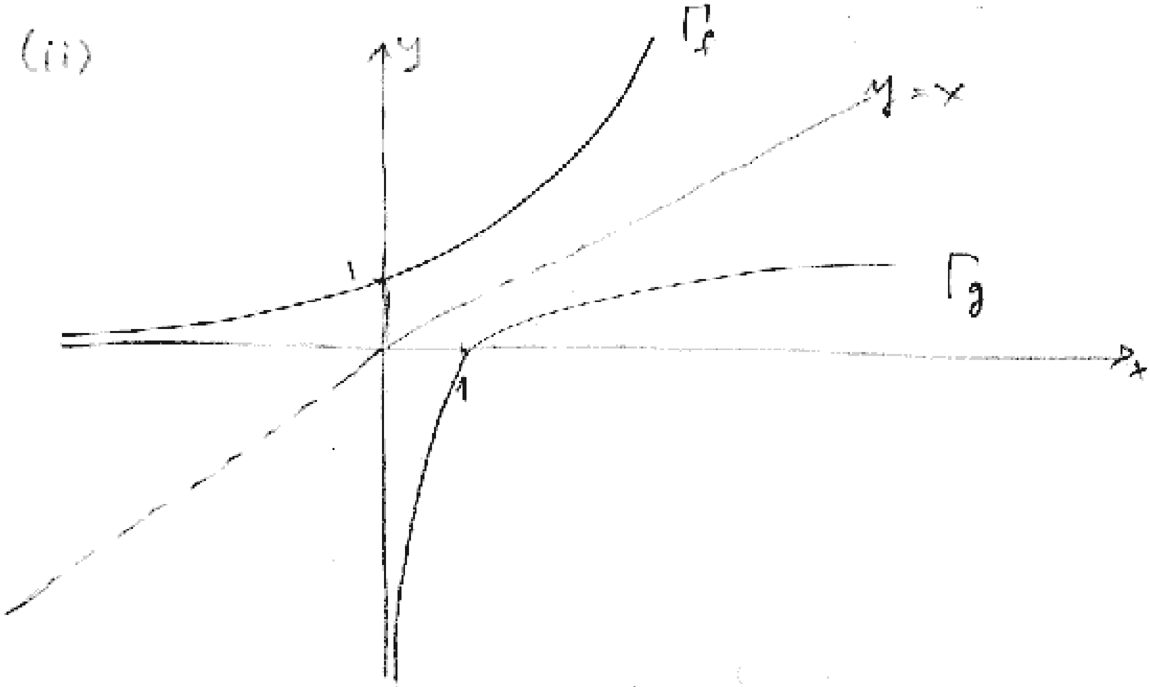
$t_2 = 10 \Rightarrow \quad 2^x = 10 \,/\log_2 \Rightarrow \quad \boxed{x_2 = \log_2 10}$

2. Zadatak 2. (i) $f(x)=5^x$, $g(x)=\log_5 x$

Veza: $5^{\log_5 x}=x$

$\log_5 5^x = x$

(ii)



f: rast $\langle -\infty, \infty \rangle = \mathbb{R}$, pad $=\emptyset$

g: rast $\langle 0, \infty \rangle$, pad $=\emptyset$

(iii) $\log_5 x \geq 1 \quad / 5^{-}$

$\boxed{x \geq 5}$ tj. $\boxed{x \in [5, \infty \rangle}$

3. (i) $f(x) = (x-1)^3 + 3$

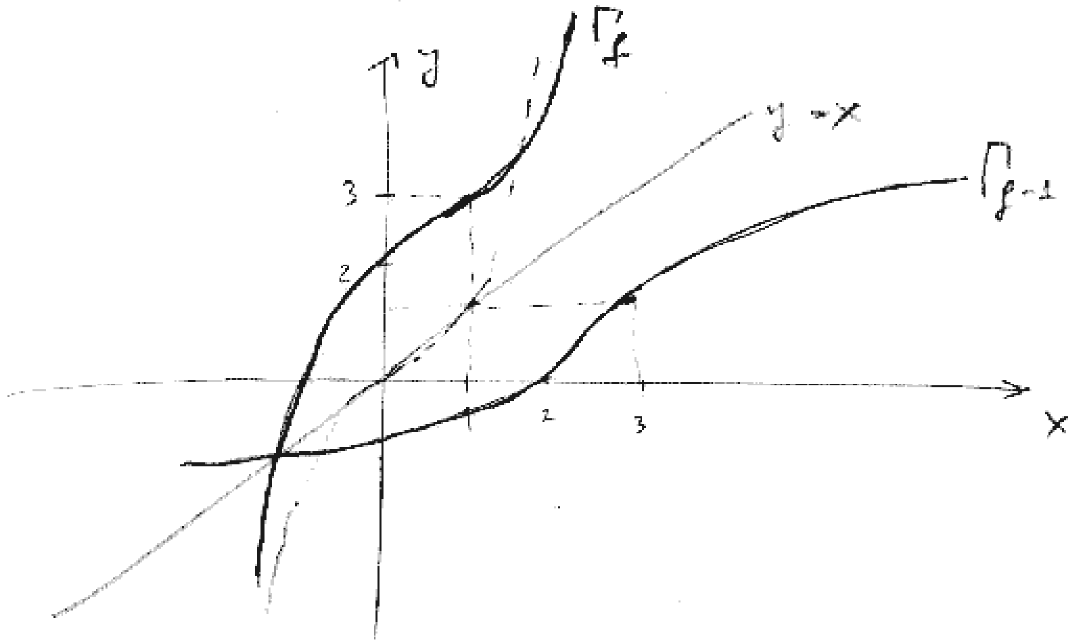
$$y = (x-1)^3 + 3$$

$$y - 3 = (x-1)^3 \quad / \sqrt[3]{\ }$$

$$\sqrt[3]{y-3} = x - 1 \Rightarrow \quad x = \sqrt[3]{y-3} + 1$$

$$\boxed{f^{-1}(x) = \sqrt[3]{x-3} + 1}$$

(ii)



(iii) f: točka infleksije je $\boxed{(1,3)}$

f^{-1}: —— ,, —— je $\boxed{(3,1)}$

4. (i) $y = -4x-1$

$4x = -y-1$

$x = -\frac{1}{4}y - \frac{1}{4}$

$$\boxed{f^{-1}(x) = -\frac{1}{4}x - \frac{1}{4}}$$

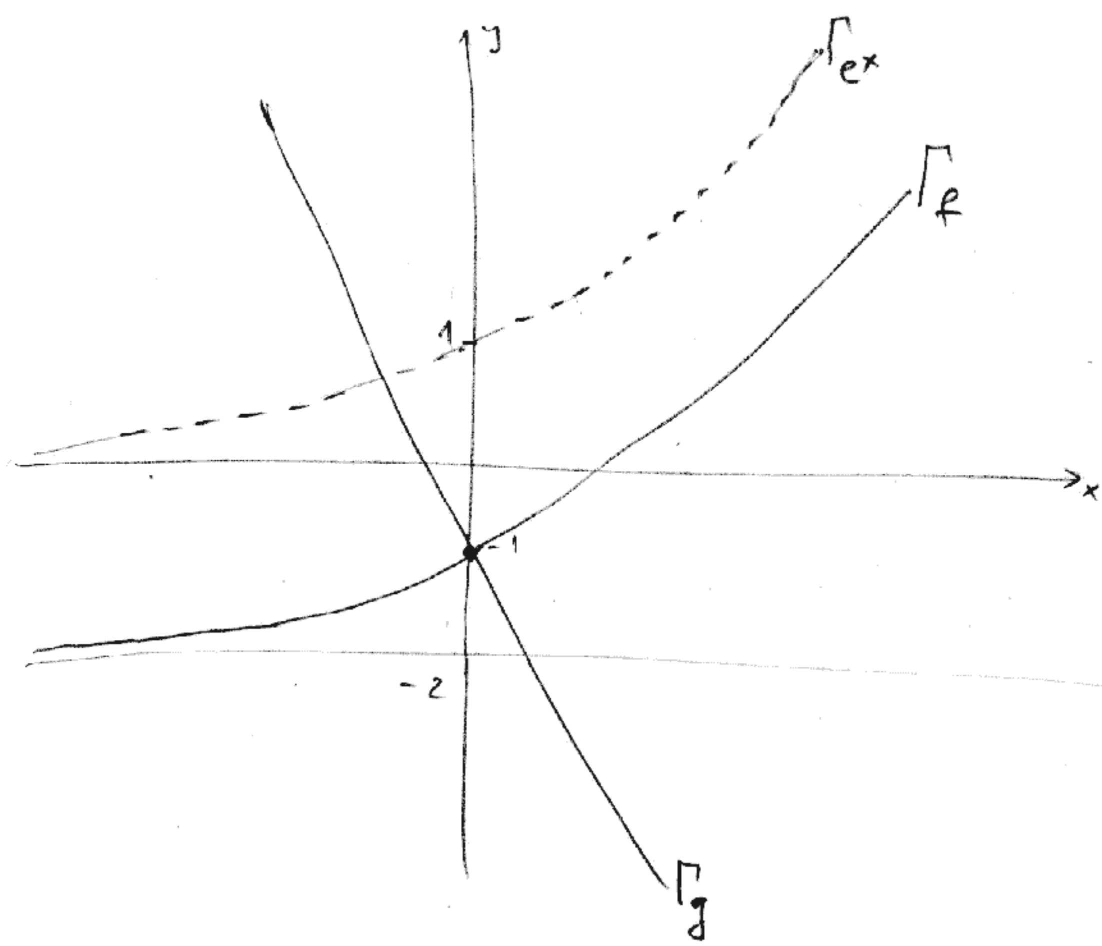
(ii) $-4x-1 = 3 \,/\, f^{-1}(x)$

$x = f^{-1}(3) = -\frac{1}{4}\cdot 3 - \frac{1}{4} = -\frac{3}{4} - \frac{1}{4} = -1$

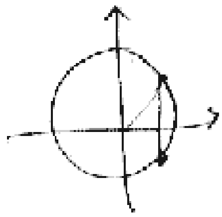
$$\boxed{x=-1}$$

(iii) $e^x - 2 = -4x-1$

$\left.\begin{array}{l} y = e^x - 2 \\ y = -4x-1 \end{array}\right\}$ pokazujemo da se ove dvije krivulje sijeku u samo 1 točki:



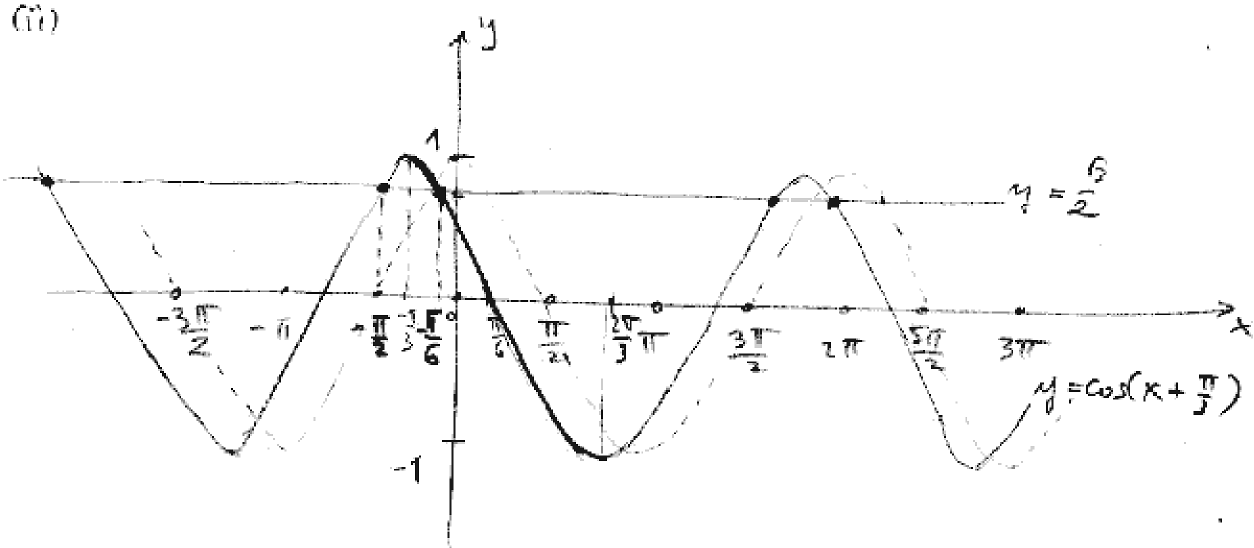
5. (i) $\cos\left(x + \frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$



$$x_1 + \frac{\pi}{3} = \frac{\pi}{6} + 2k\pi,\ k \in \mathbb{Z}$$
$$x_2 + \frac{\pi}{3} = -\frac{\pi}{6} + 2k\pi,\ k \in \mathbb{Z}$$

$$\Rightarrow \boxed{\begin{array}{l} x_1 = -\frac{\pi}{6} + 2k\pi,\ k \in \mathbb{Z} \\ x_2 = -\frac{\pi}{2} + 2k\pi,\ k \in \mathbb{Z} \end{array}}$$

(ii)



(iii) Možemo uzeti npr. interval $\underline{\left[-\frac{\pi}{3}, \frac{2\pi}{3}\right]}$ (vidi sliku!)